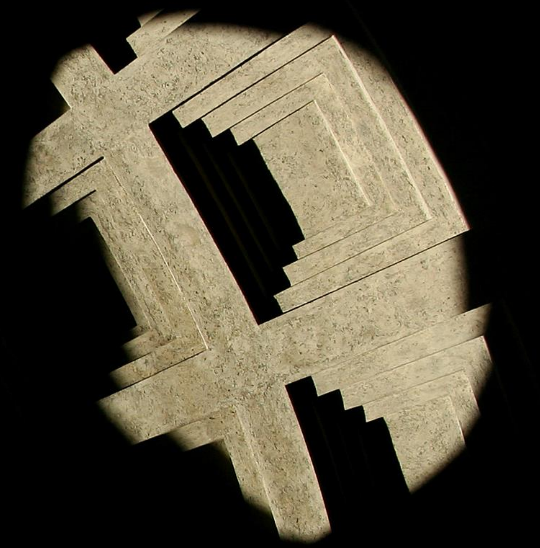
Solar Geometry



Penn State University (PSU)
Engineering Go For It (eGFI)

Sponsors

This lesson on solar geometry is possible due to the generosity of ProjectCANDLE and CarbonEARTH. You can visit their respective websites via the following links:

http://www.engr.psu.edu/candle/

http://www.carbonearth.org

Introduction / Motivation

This module is dedicated to understanding the geometrical relationship between the Earth and the Sun and learning how to describe the relationship mathematically. By the end of this module you should understand:

1. The tilt of the Earth relative to the Earth's orbit around the sun
2. The influence of this tilt and orbit on seasons at various locations on Earth
3. The location of the sun throughout different times of the year
4. How time (year, day, hour) and position (longitude and latitude) are quantified on Earth
5. Time zones
6. How to calculate the position of the sun at any time and location on Earth

This module is split into 3 major sections, each of which has a worksheet (with key) and an activity (with key). To the teachers discretion, a quiz may accompany each section. A quiz template has been provided. The three sections are:

I. Understanding the Earth-Sun relationship
II. Quantifying time and position on Earth
III. Solar position (calculating solar angles)

Markers

Various markers will be used throughout this lesson to indicate important aspects of the presentation such as; when a question should be posed, when extra caution should be exercised, or when an activity should be performed. Markers are as follows:

This symbol indicates a question and/or Discussion

This symbol indicates when additional caution should be exercised

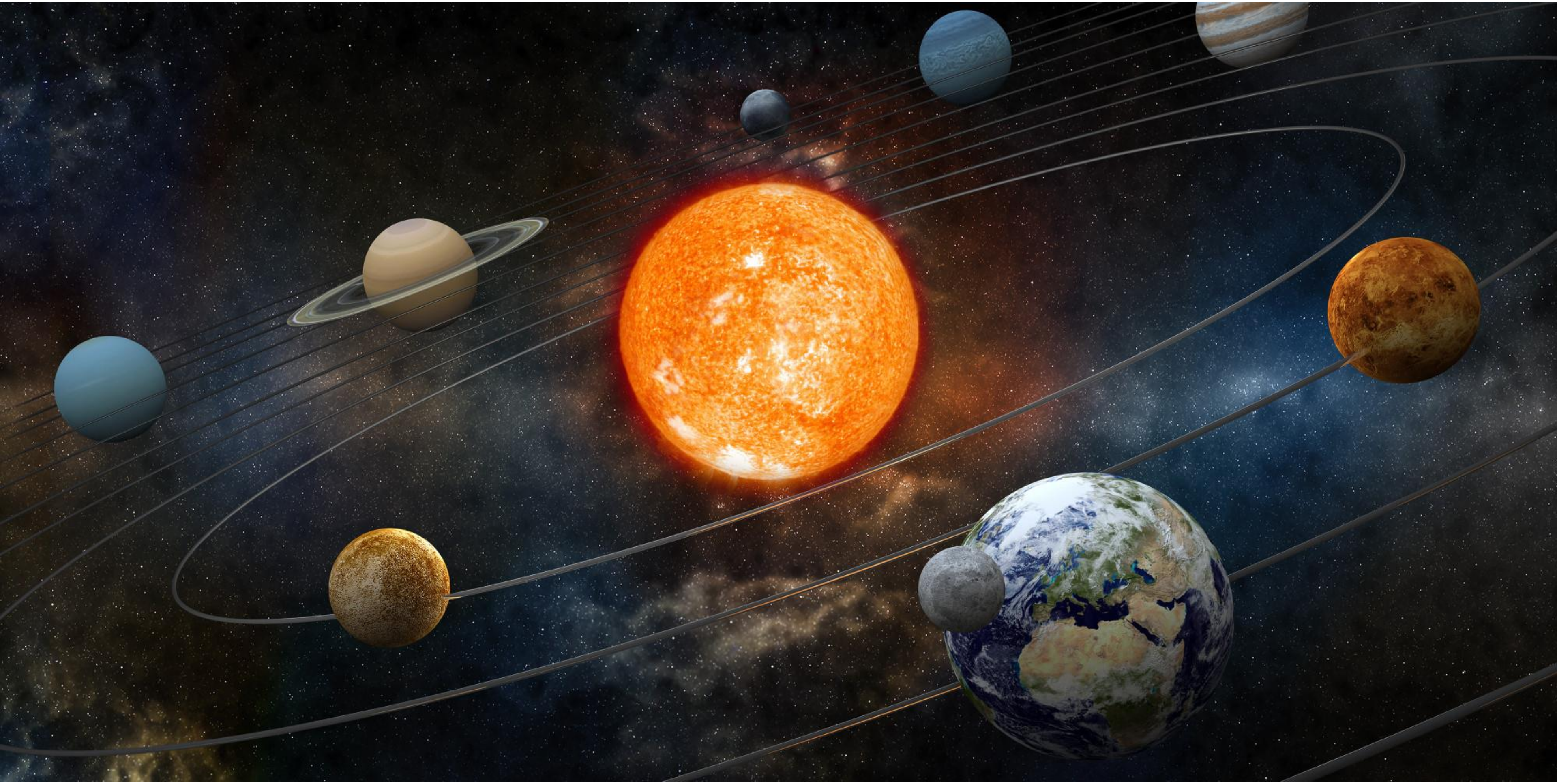
This symbol indicates an activity!

This symbol indicates a worksheet. There is one worksheet per section

This symbol indicates a quiz. There is one quiz per section

SECTION I

Understanding the Earth-Sun relationship



What do you know?

Class discussion

Use the space below to document the students' current knowledge of solar geometry:

Videos

Mechanism of the Seasons
http://www.youtube.com/watch?v=WLRA87TKXLM

Physical Science 9.2a - The Earth Moon Sun
http://www.youtube.com/watch?v=FjCKwkJfg6Ym

Physical Science 9.2b - Rotation and Revolution
http://www.youtube.com/watch?v=op6vsLNf3WY

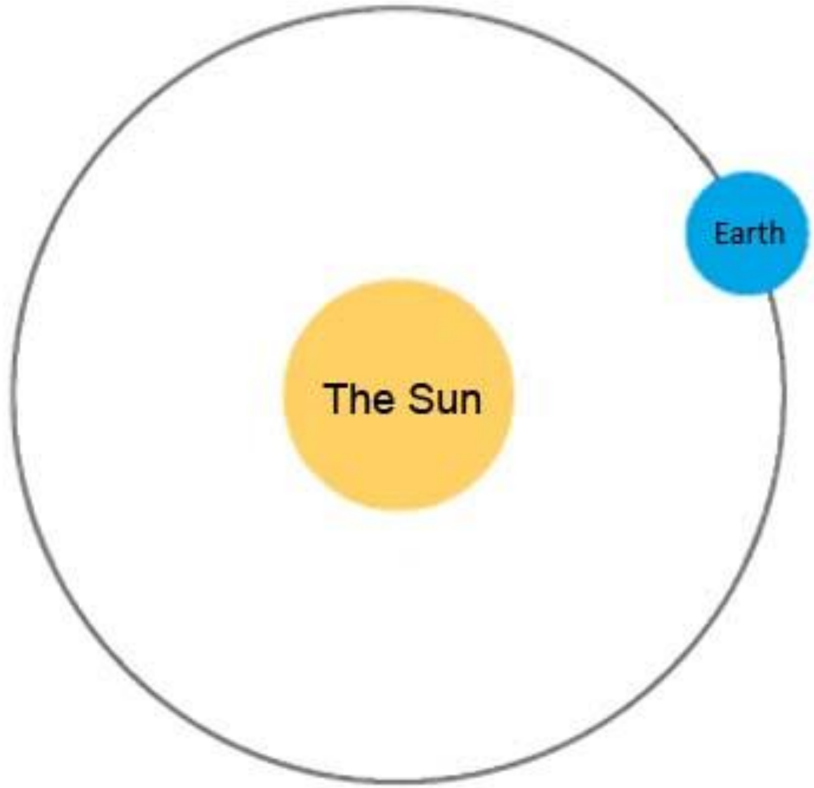
Spaceship Earth - An animated documentary of how Earth works 1/52
http://www.youtube.com/watch?v=JaG70cJ8vDE

As you watch these videos, think about the following:

Q: In which direction does the Earth orbit the sun?
Q: In which direction does the Earth rotate about its own axis?
Q: What is the Earth’s axial tilt? (relative to its orbital plane)
Q: What causes the seasons on earth?
Q: How does time of year effect length of day?
Q: How do we technically define: year, day, hour.

Earth’s Orbit

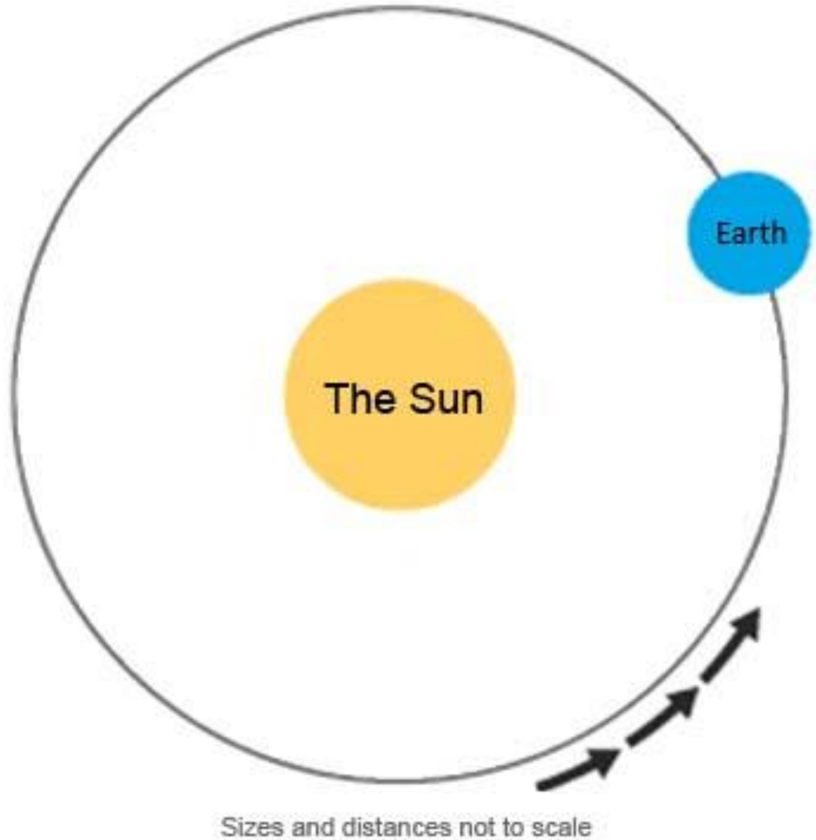
When viewing the solar system from above (“*Plan View*”), in which direction does the **EARTH** rotate around the **SUN**?



Sizes and distances not to scale

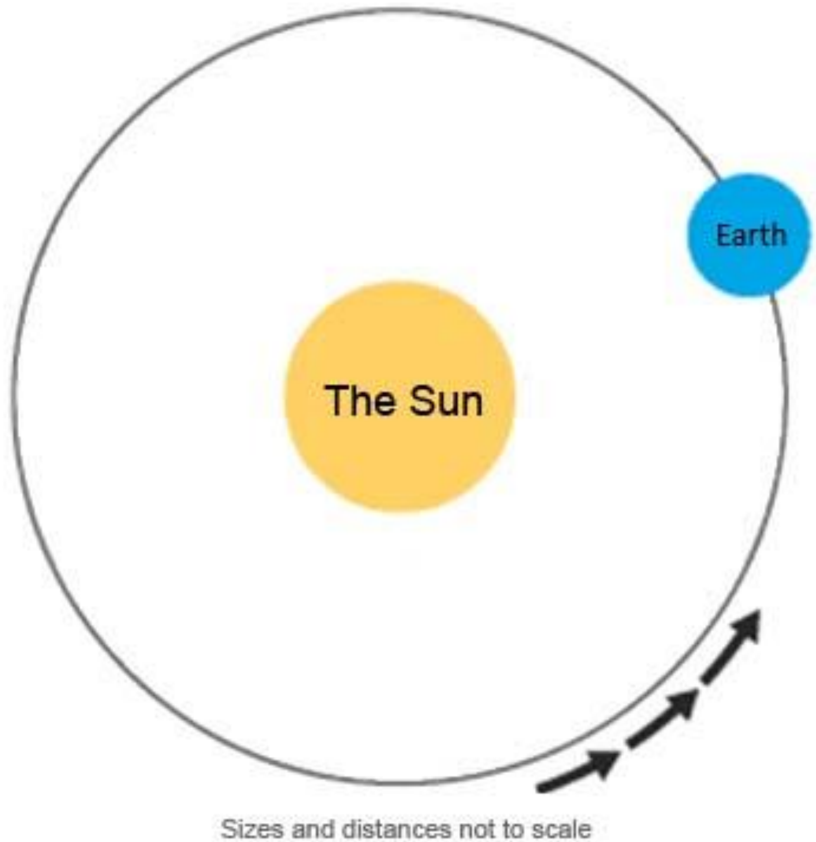
Earth's Orbit

Answer: Earth orbits **counterclockwise** around the sun (when viewed from above)



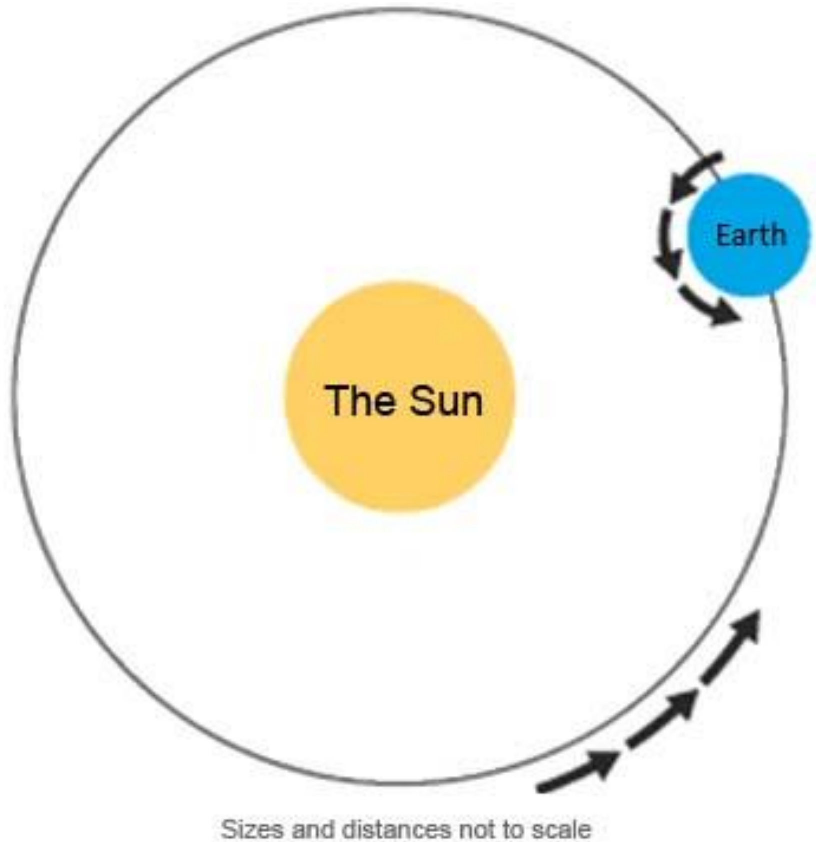
Earth’s Rotation

When viewed from above (“*Plan View*”), in which direction does the **EARTH** rotate around **its own axis**?



Earth's Rotation

Answer: Earth rotates **counterclockwise** around its own axis (when viewed from above)

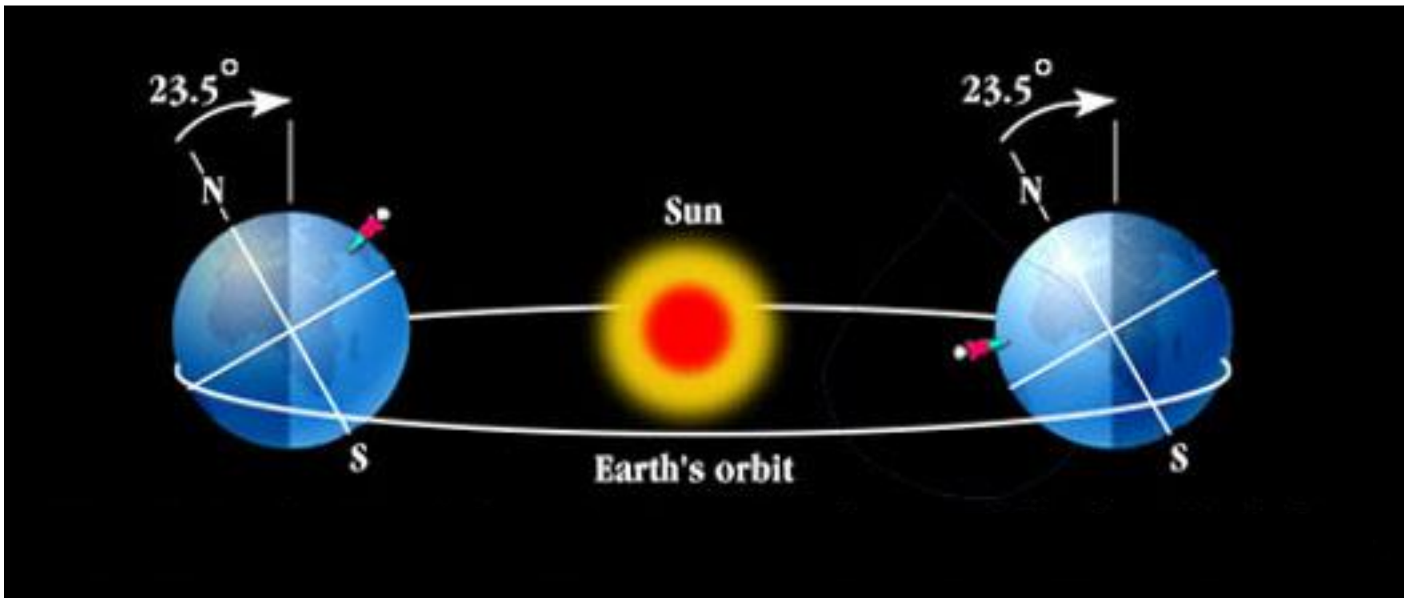


Earth’s tilt

What is the Earths axial tilt relative to its orbital plane?

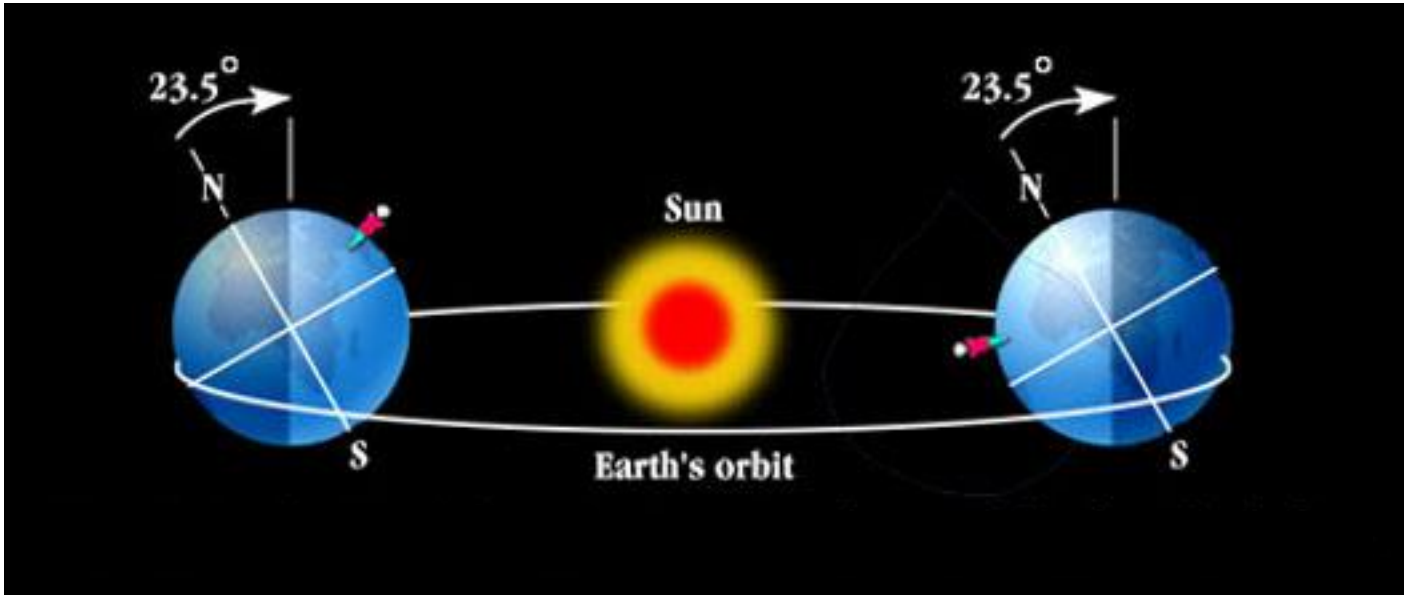
Earth’s tilt

Answer: Earth is tilted off-axis **23.5°** relative to its own orbital plane



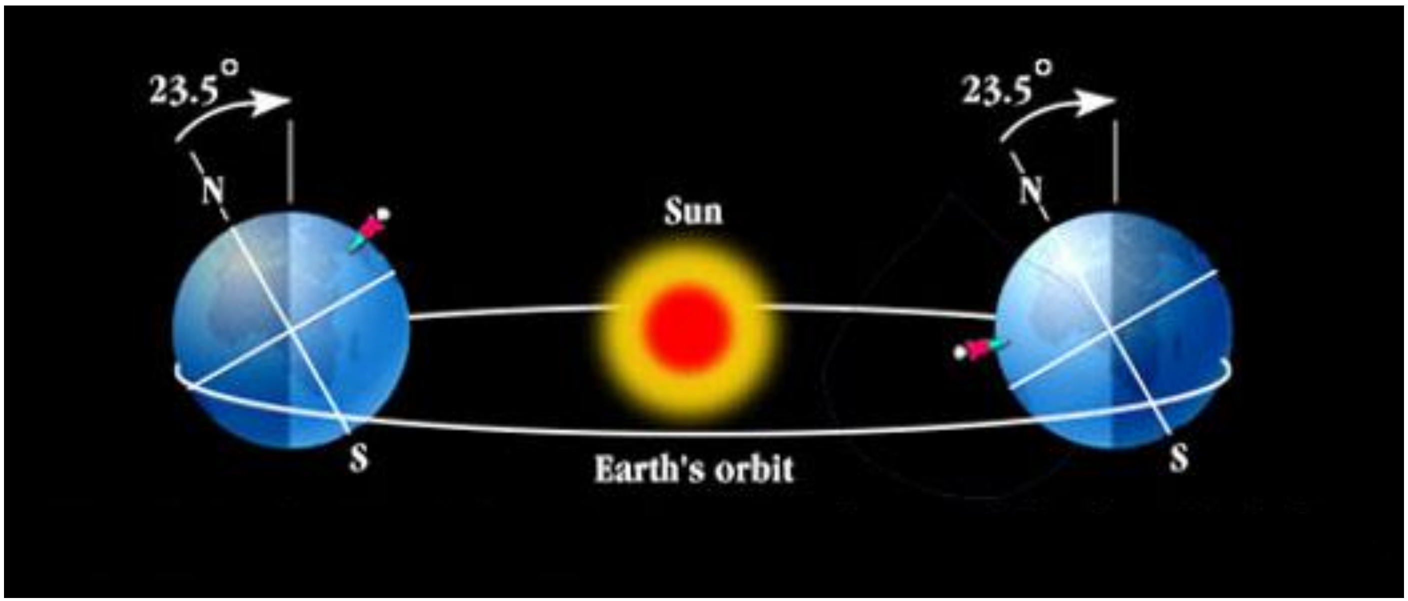
Earth's tilt

What effect, if any, does this axial tilt have on our experience on earth?



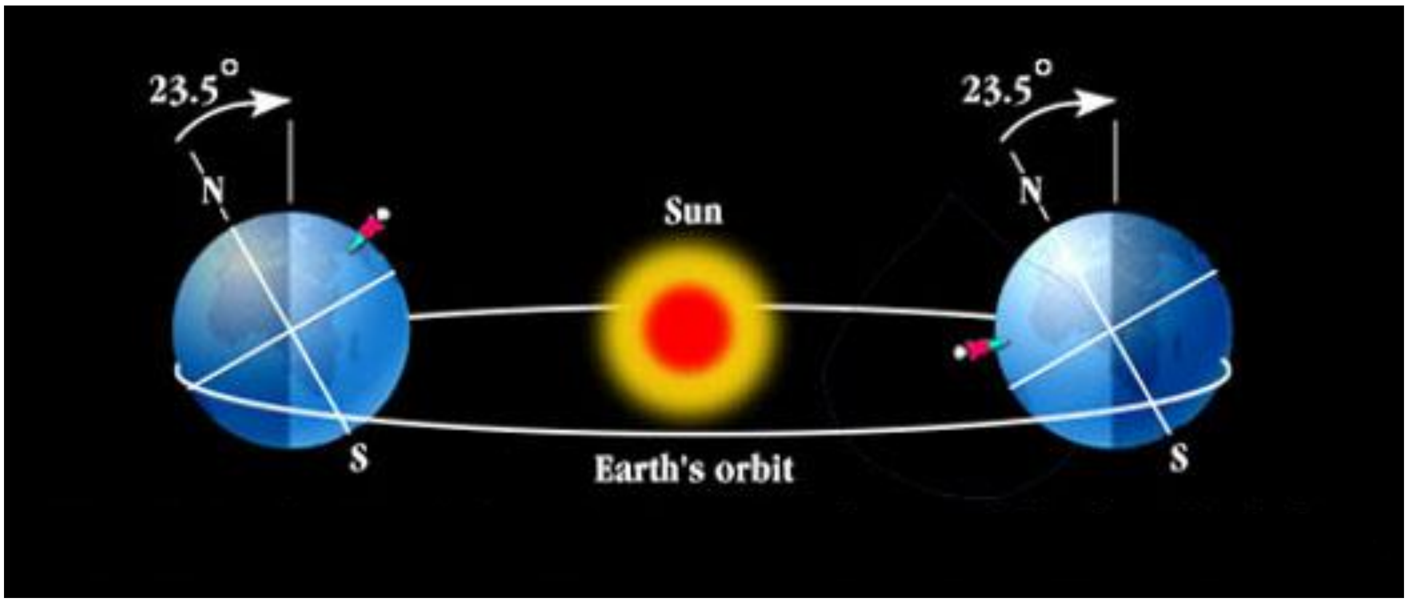
Earth’s tilt

Earths axial tilt causes the **seasons**!



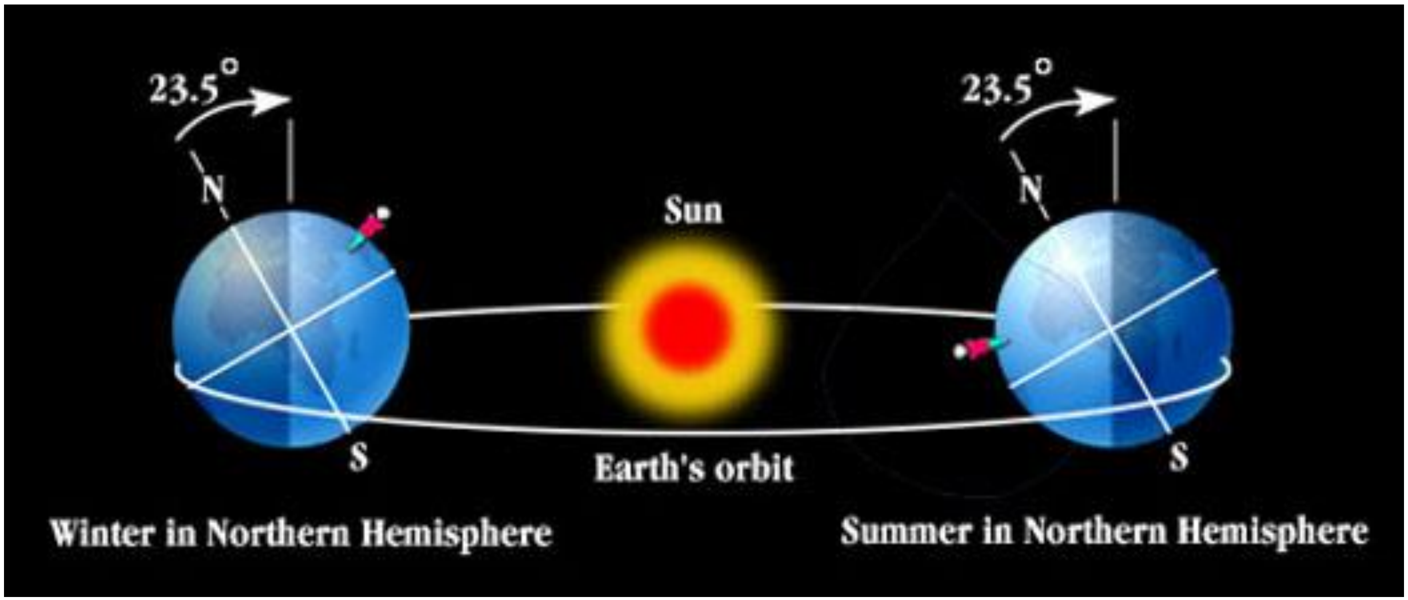
Earth’s tilt

In the diagram below, which season are represented?



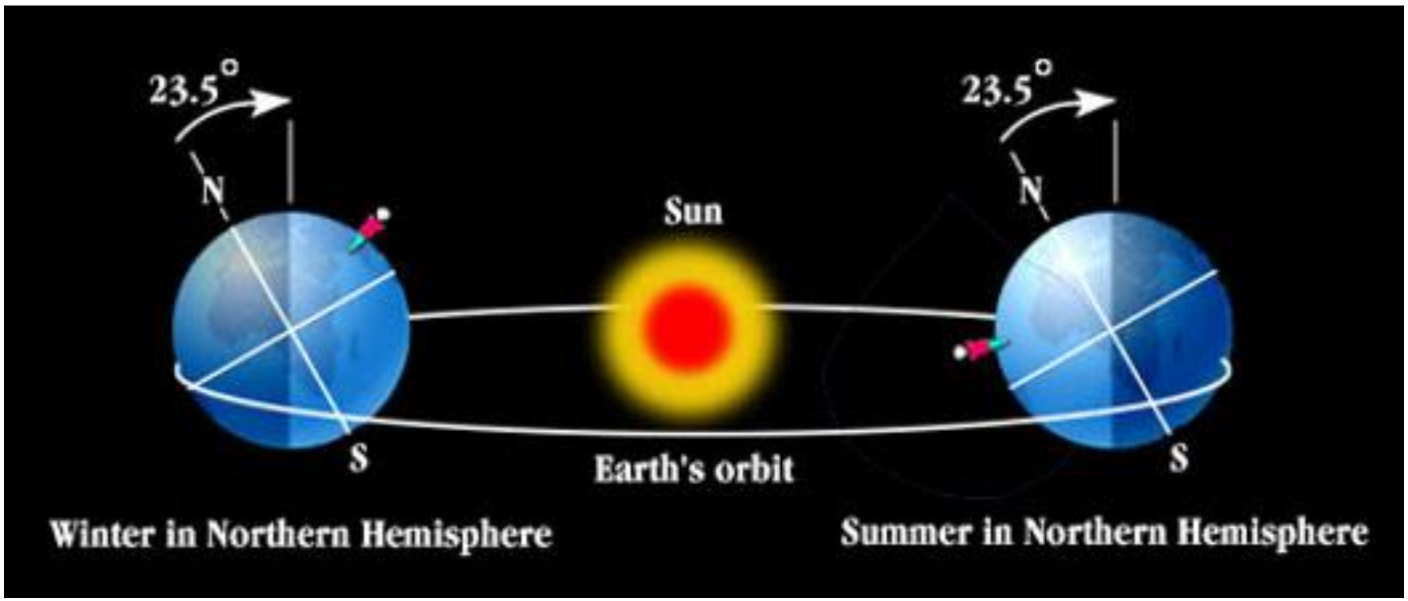
Earth’s tilt

The earth on the left, is winter in the *northern hemisphere* because the earth is tilted **away** from the sun. The earth on the right is summer in the *northern hemisphere* because the earth is tilted **toward** the sun.



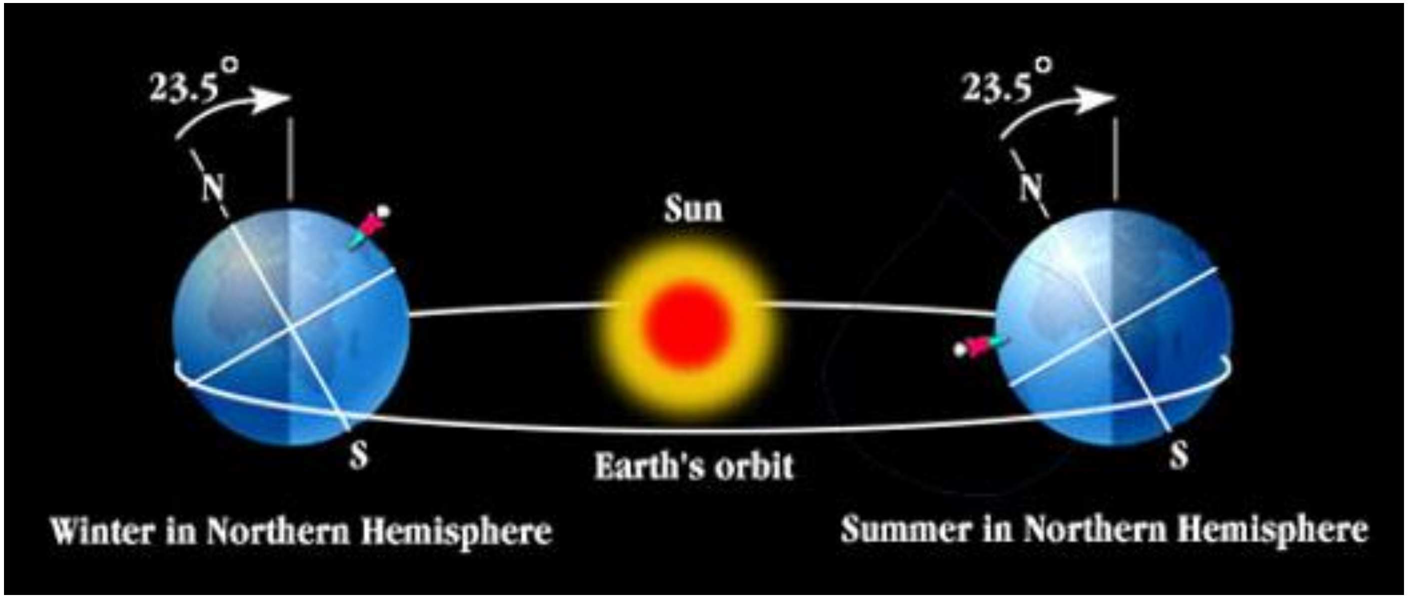
Earth’s tilt

Why must we make the distinction of *“northern hemisphere?”*



Earth's tilt

Answer: because the seasons are reversed in the southern hemisphere! That is, when the northern hemisphere has summer (sun highest in the sky), the southern hemisphere has winter (sun lowest in the sky).



Key Terms

- **Year**
- **Day**
- **Hour**
- **Julian Day**
- **Summer Solstice**
- **Winter Solstice**
- **Vernal (Spring) Equinox**
- **Autumnal (Fall) Equinox**

Key Terms (defined)

Year the amount of *time* it takes for the earth to complete one complete orbit around the sun (approximated 365 days)

Day the amount of *time* it takes for the earth to complete one full rotation about it's own axis (approximately 24 *hours*)

Hour the amount of *time* it takes for a fixed point on earth to rotate through 15° (360° / 24 hours)

Julian Day the whole number integer assigned to each day as it falls chronologically throughout the year. That is, the range of Julian day is from 1 – 365. For example, March 23 = 31 (Jan) + 28 (Feb) + 23 (March) = 82

Key Terms (defined)

Summer Solstice	the time of the year when the sun reaches its highest position in the sky (in the northern hemisphere). This occurs on June 21/22
Winter Solstice	the time of the year when the sun reaches its lowest position in the sky (in the northern hemisphere). This occurs on December 21/22.
Vernal (Spring) Equinox	the period of the year (following summer) when all places on earth receive equal amounts of daylight and night. This occurs around March 20.
Autumnal (Fall) Equinox	the period of the year (following winter) when all places on earth receive equal amounts of daylight and night. This occurs around September 22.

Activity

Draw (in plan view) the earth's position in relation to the sun during the following four times of the year and indicate their Julian Day:

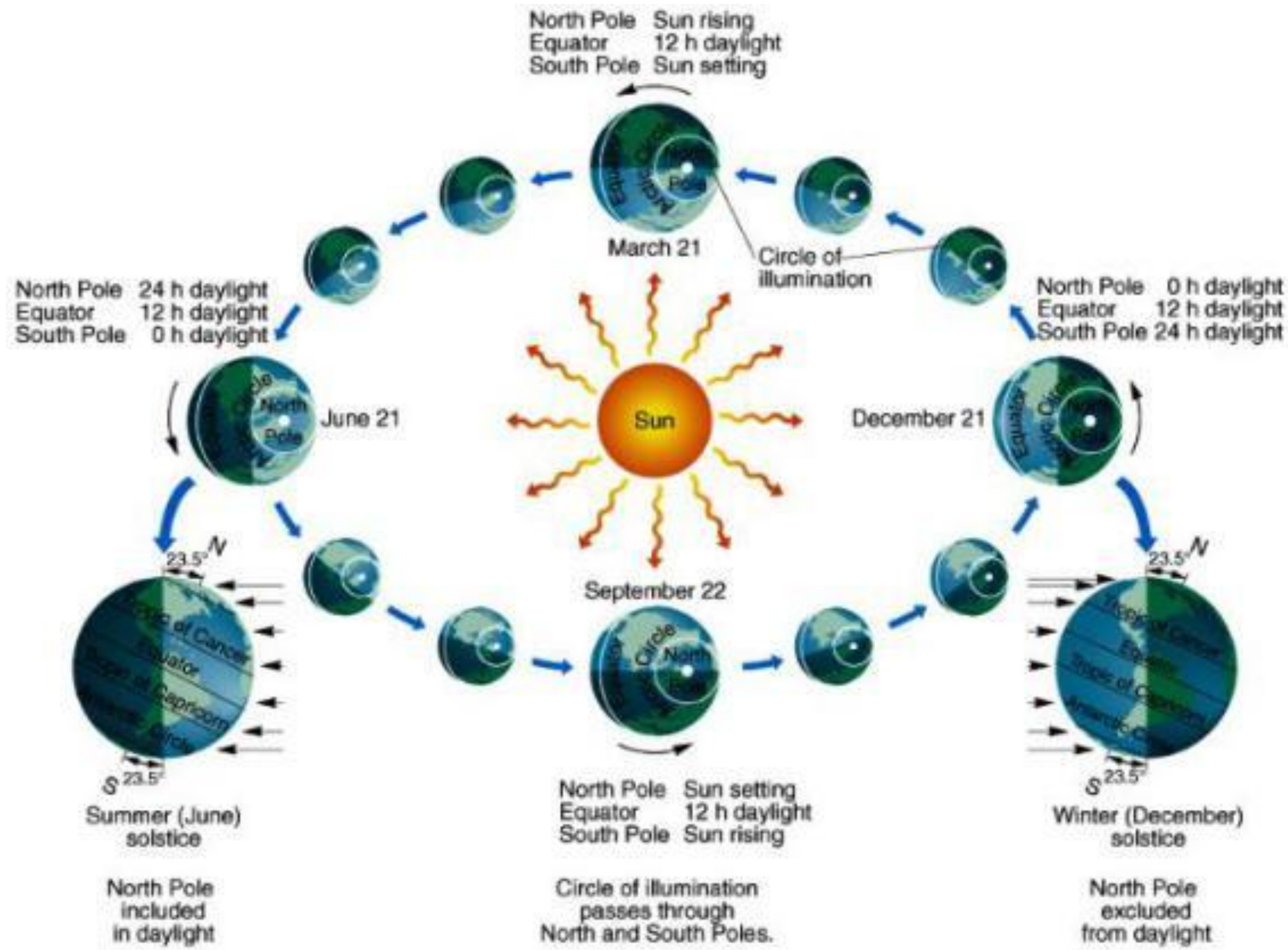
- June 22 (Summer Solstice)
- September 23 (Autumnal Equinox)
- December 22 (Winter Solstice)
- March 21 (Vernal Equinox)

Additionally, indicate the Earths orbital position on your birthday and calculate the Julian Day for your birthday!

For reference, the number of days in each month are provided below:

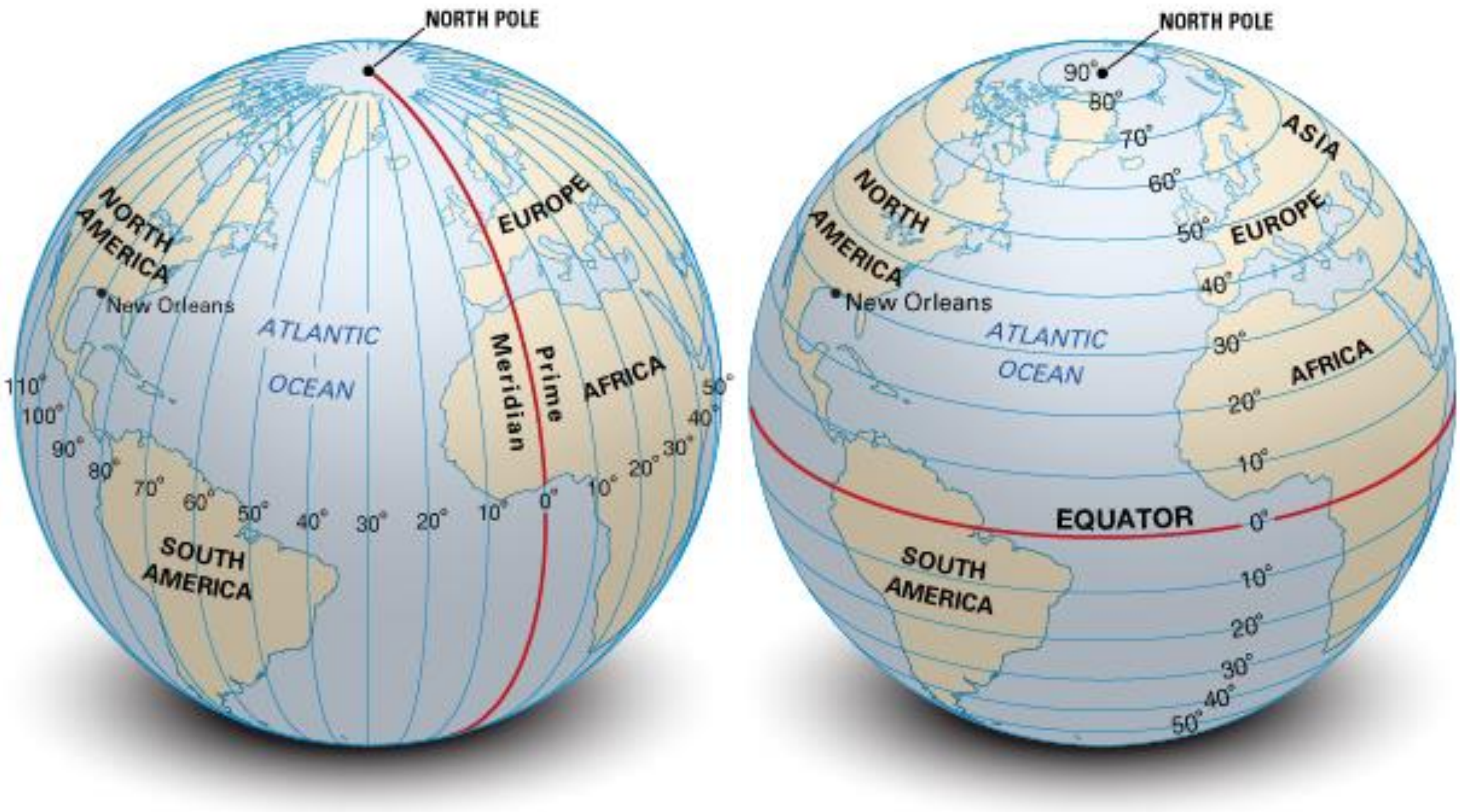
Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
# of days	31	28	31	30	31	30	31	31	30	31	30	31

Quiz


North Pole Sun rising
Equator 12 h daylight
South Pole Sun setting
March 21
Circle of illumination
North Pole 24 h daylight
Equator 12 h daylight
South Pole 0 h daylight
June 21
Sun
December 21
North Pole 0 h daylight
Equator 12 h daylight
South Pole 24 h daylight
September 22
Summer (June) solstice
North Pole included in daylight
North Pole Sun setting
Equator 12 h daylight
South Pole Sun rising
Circle of illumination passes through North and South Poles.
Winter (December) solstice
North Pole excluded from daylight

SECTION II

Quantifying **time** and **position** on **EARTH**



Videos

Latitude and Longitude
https://www.youtube.com/watch?v=swKBi6hHHMA

How the International Date Line Works
http://www.youtube.com/watch?v=hPpWCTHjzQI

Understanding Time Zones
http://www.youtube.com/watch?v=X1DkiuaFCuA

Animation Explaining the International Date Line | Video
http://www.youtube.com/watch?v=m0QOlFIZKXI

As you watch these videos, think about the following:

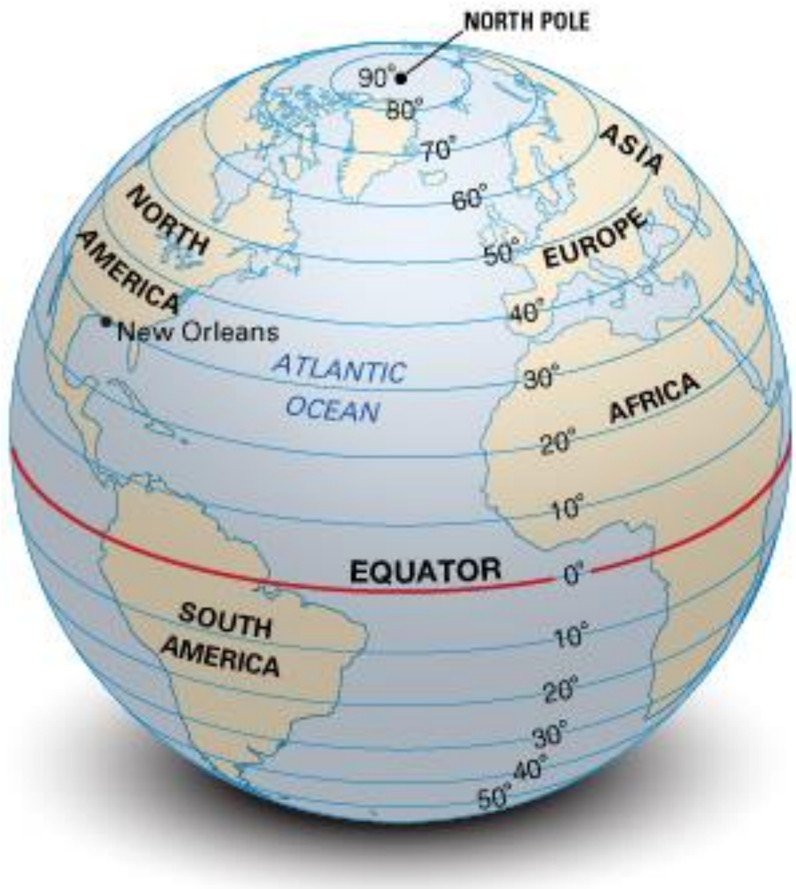
Q: What is the shape of the earth?
Q: What is the purpose of latitude and longitude?
Q: How do we describe location on earth?
Q: How do these imaginary lines relate to keeping time on Earth?
Q: Where does a new day begin?
Q: What is the international date line?

Latitude

What is latitude?

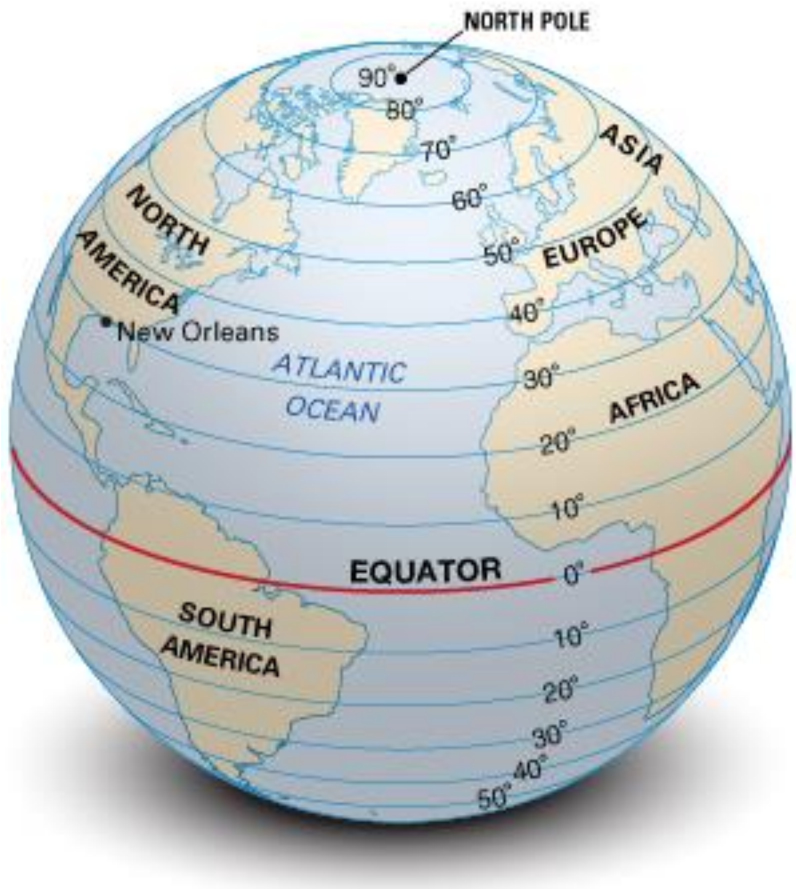
Latitude

Latitude is a geographical coordinate that specifies the **north-south** position of a point on the Earth's surface.



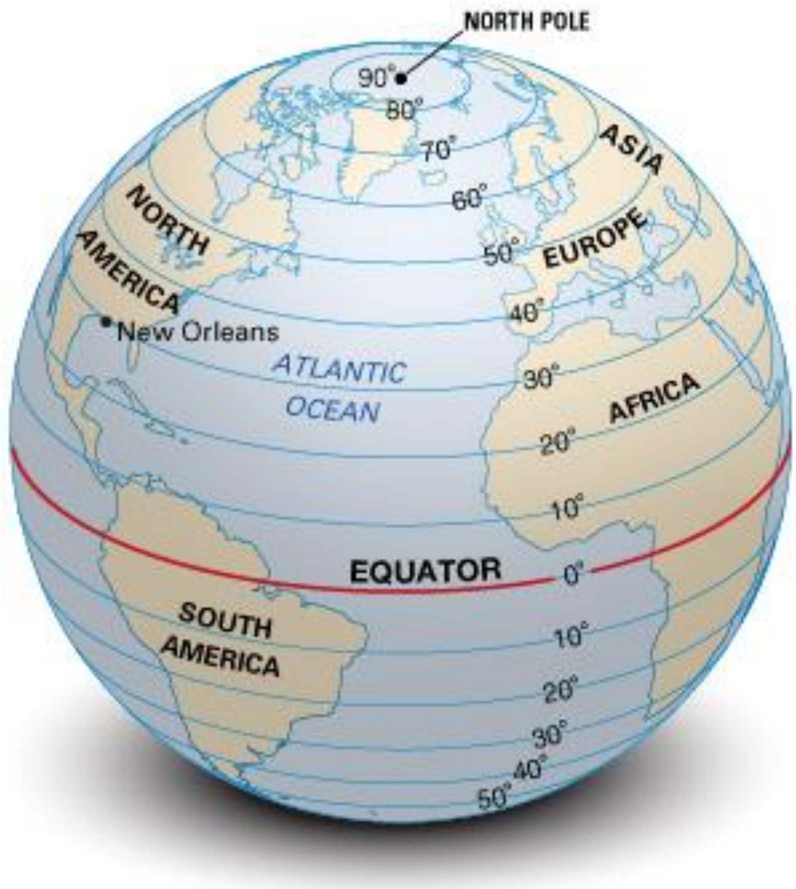
Latitude

Lines of constant **latitude** run east-west and are measured in degrees. Latitude ranges from -90° (South Pole) to +90° (North Pole) [$-90° \leq l \leq +90°$]



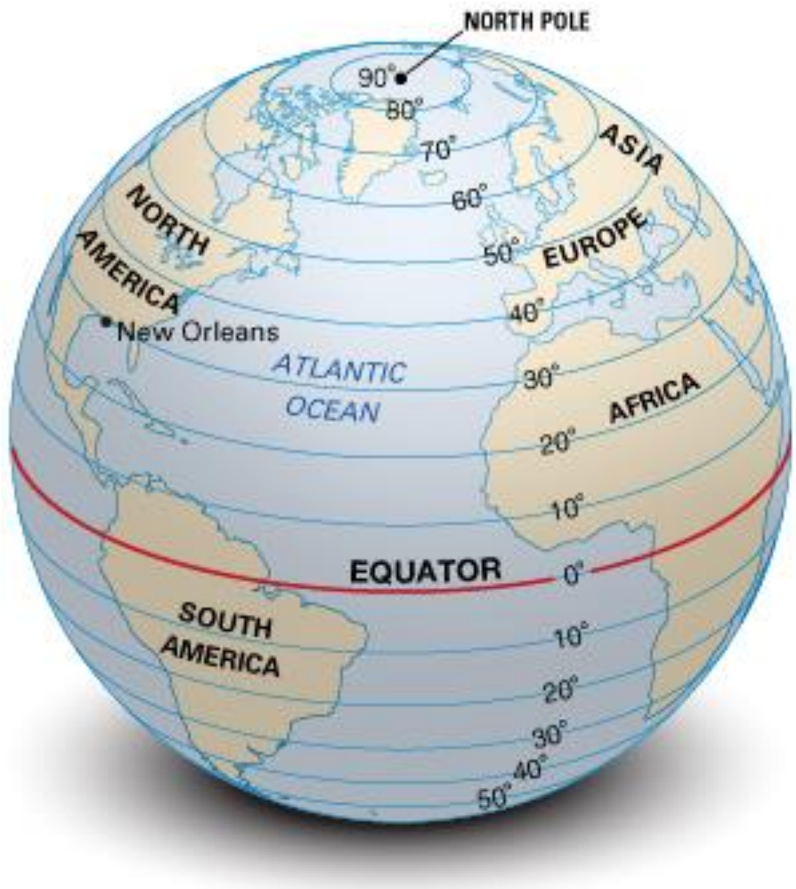
Latitude

Where is the logical place for the lines of 0° *latitude*?



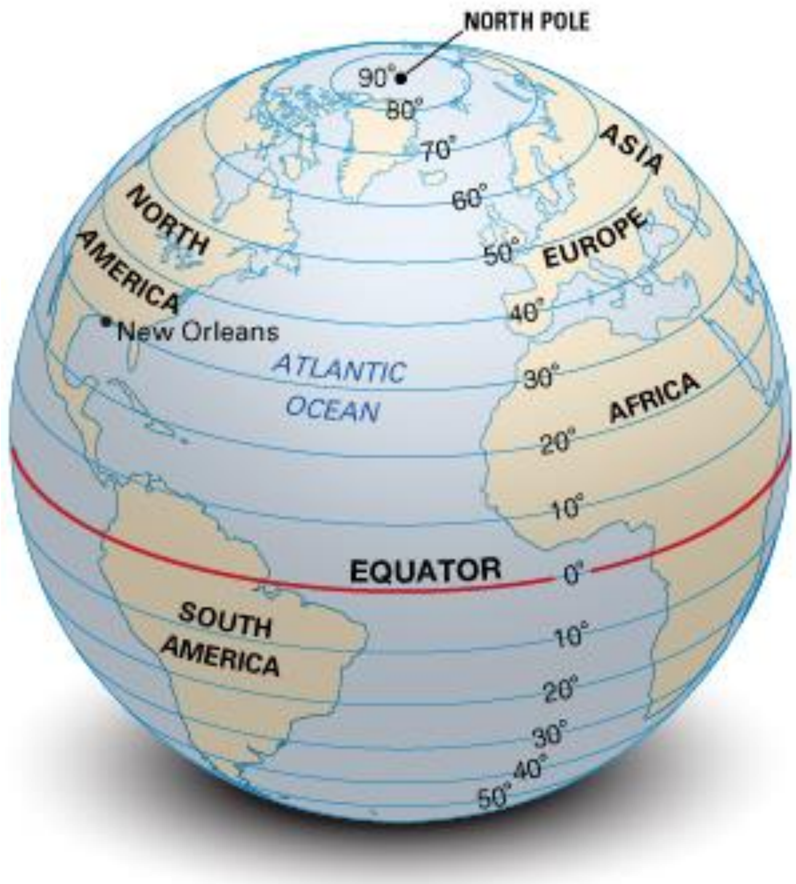
Latitude

The equator!



Latitude

Facts about **latitude:**



- Are known as "parallels"
- Run in an east-west direction
- Measure distance north and south from the equator
- Are parallel to one another and never meet
- Cross the prime meridian at right angle (more on this next)
- Lie in the planes that cross the Earth's axis as right angles
- Get shorter toward the poles, with the equator as the largest circle

Longitude

What is longitude?

Longitude

Longitude is a geographical coordinate that specifies the **east-west** position of a point on the Earth's surface.

Longitude

Lines of constant **longitude** run north-south and are measured in degrees. Longitude ranges from -180° to +180° [-180° ≤ L ≤ +180°]

Longitude

Where is the logical position for 0° longitude?

Longitude

As is turns out, there is no logical location of 0° longitude. 0° longitude (also called the “Prime meridian”) passes through the Royal Observatory in Greenwich, England. This is for historical reasons.

Longitude

Lines of positive longitude are **east** of the prime meridian
Lines of negative longitude are **west** of the prime meridian

Longitude

Facts about **longitude:**

- Are known as “meridians”
- Run in a north-south direction
- Measure distance east or west of the prime meridian
- Are farthest apart at the equator and meet as the poles
- Cross the equator at right angles
- Lie in the planes that pass through the Earth’s axis
- Are equal in length

Time zones

Besides being 0° latitude, what other significance do you think the prime meridian has?

Time zones

Besides being 0° latitude, what other significance do you think the prime meridian has?

The prime meridian is used as the primary *time standard* by which the whole world regulates their clocks and time! This is called **Coordinated Universal Time (UTC)** and the time zone at the prime meridian is denoted UTC-00:00

UTC-00:00

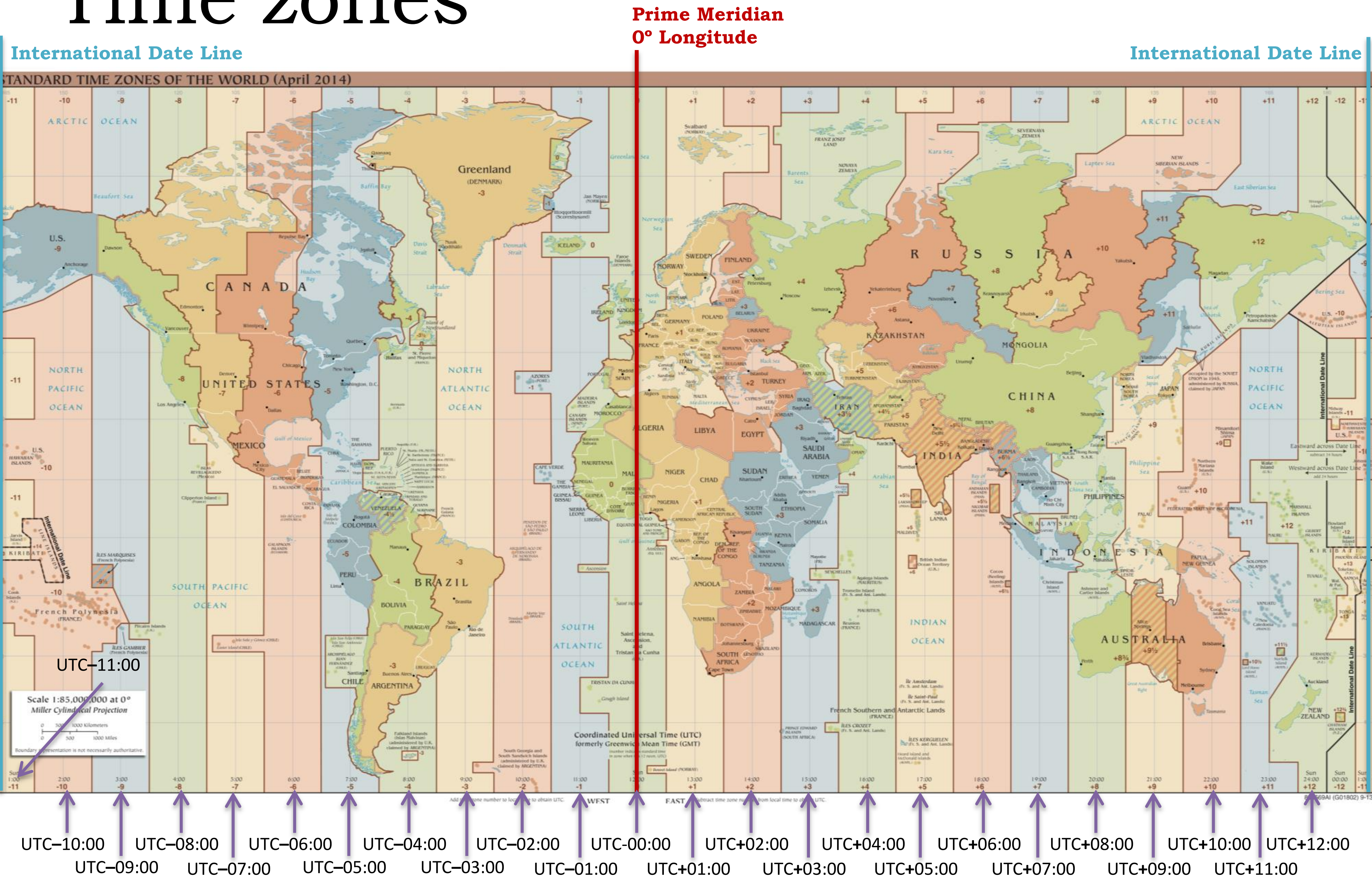
Designates the offset of a time zone from Coordinated Universal time (i.e. the time in Greenwhich, England)

Stands for "Coordinated Universal Time"

Time zones to the **east** of the prime meridian are offset by a positive number. That is, 2pm at UTC-00:00 is 3pm at UTC+01:00

Time zones to the **west** of the prime meridian are offset by a negative number. That is, 2pm at UTC-00:00 is 1pm at UTC–01:00

Time zones



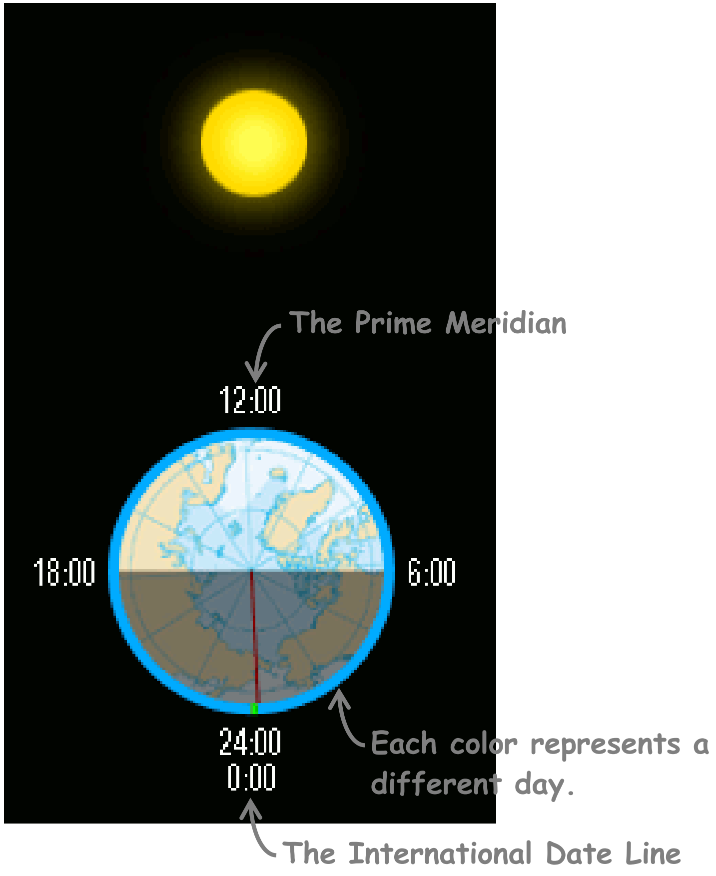
Prime Meridian
0° Longitude
International Date Line
International Date Line
STANDARD TIME ZONES OF THE WORLD (April 2014)
UTC–11:00
UTC–10:00
UTC–09:00
UTC–08:00
UTC–07:00
UTC–06:00
UTC–05:00
UTC–04:00
UTC–03:00
UTC–02:00
UTC–01:00
UTC-00:00
UTC+01:00
UTC+02:00
UTC+03:00
UTC+04:00
UTC+05:00
UTC+06:00
UTC+07:00
UTC+08:00
UTC+09:00
UTC+10:00
UTC+11:00
UTC+12:00

International Date Line

What significance does the “International Date Line have?” What important feature do you notice about it (in relation to the prime meridian)?

International Date Line

The International Date Line is 180° around the earth from the prime meridian. The International Date Line is the point on Earth where a new calendar day begins!



Key Terms

Longitude	**International Date Line**
Latitude	**Southern Hemisphere**
Prime Meridian	**Northern Hemisphere**
North Pole	**Coordinated Universal Time (UTC)**
South Pole	
Time Zone	

Key Terms (defined)

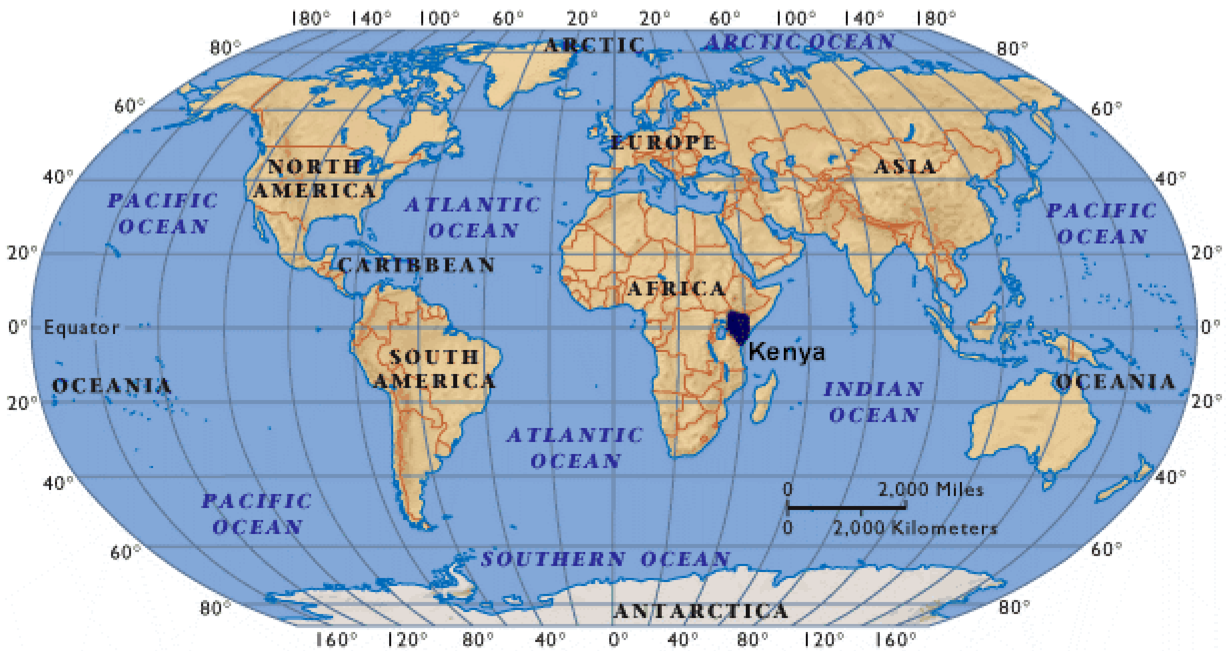
Latitude	is a geographical coordinate that specifies the north-south position of a point on the Earth's surface. Lines of constant latitude run east-west and are measured in degrees. Latitude ranges from -90° (South Pole) to +90° (North Pole). [-90° ≤ l ≤ +90°]
Longitude	is a geographical coordinate that specifies the east-west position of a point on the Earth's surface. Lines of constant longitude run north-south and are measured in degrees. Longitude ranges from -180° to +180° [-180° ≤ L ≤ +180°]
Prime Meridian	the prime meridian is recognized as the line of 0° longitude and runs north-south through the Royal Observatory in Greenwich, London
Northern Hemisphere	is the half of a planet that is north of Earth's equator.
Southern Hemisphere	is the half of a planet that is south of Earth's equator.

Key Terms (defined)

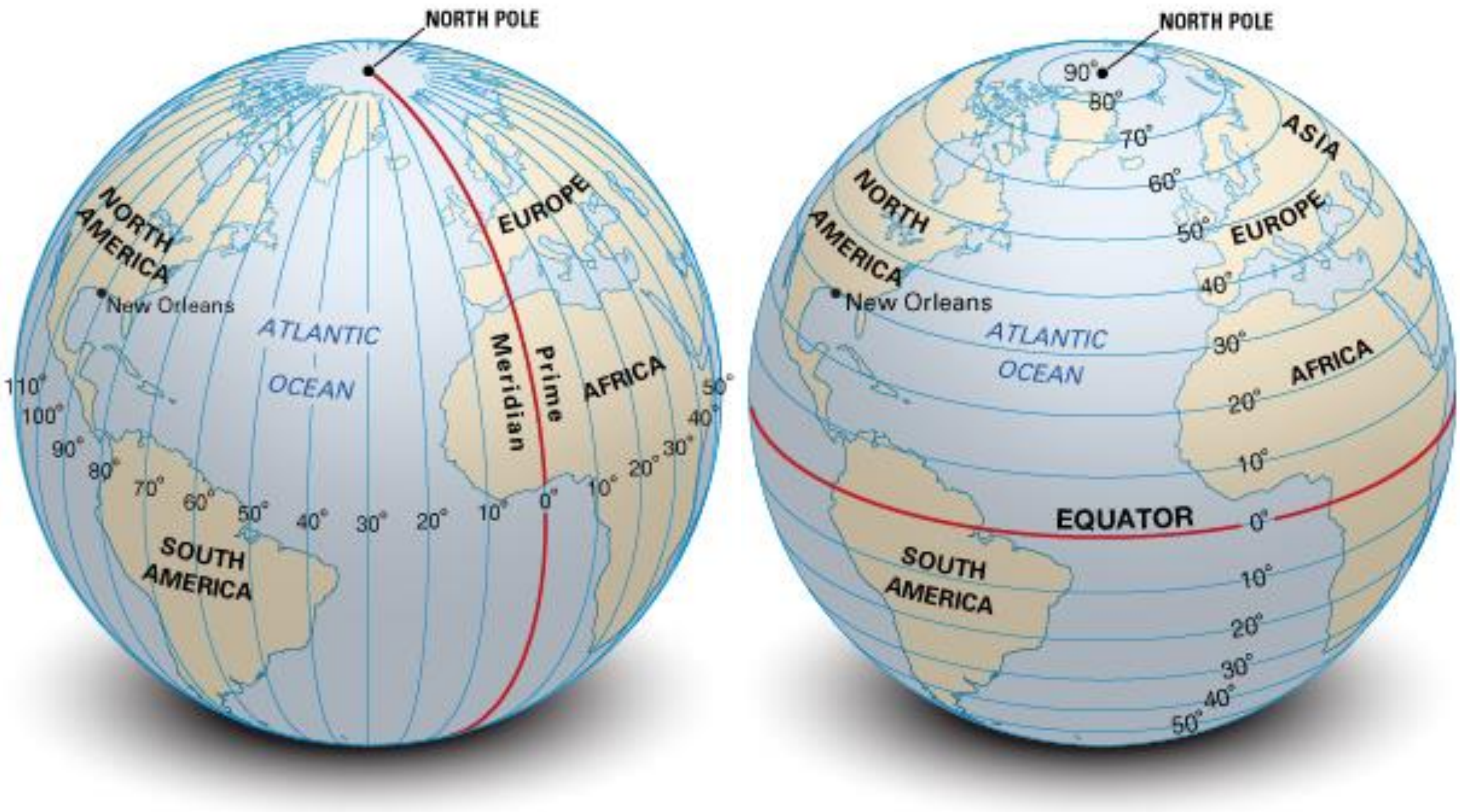
North Pole	is the northern most point in the Northern Hemisphere where the Earth’s axis of rotation meets the Earth’s surface.
South Pole	is the southernmost point on the surface of the Earth in the Southern Hemisphere. It lies on the opposite side of the Earth from the North Pole and is the other location where the Earth’s axis of rotation meets the Earth’s surface.
Coordinated Universal Time (UTC)	is the primary standard by which the whole world regulates their clocks and time! It is located at the prime meridian and is denoted as UTC-00:00.
Time Zone	A time zone is a region (of Earth) that has a uniform standard time for legal, commercial, and social purposes. There are (approximately) 12 time zones east of the prime meridian (1 per 15° of rotation) and 12 time zones west of the prime meridian (1 per 15° of rotation).
International Date Line	is the location on Earth where a calendar day begins and it located 180° around the Earth from the prime meridian.

Activity

Together, longitude and latitude form a coordinate system to quickly and easily identify a position on Earth. The goal of this activity is to familiarize students with using longitude and latitude to locate places on Earth.



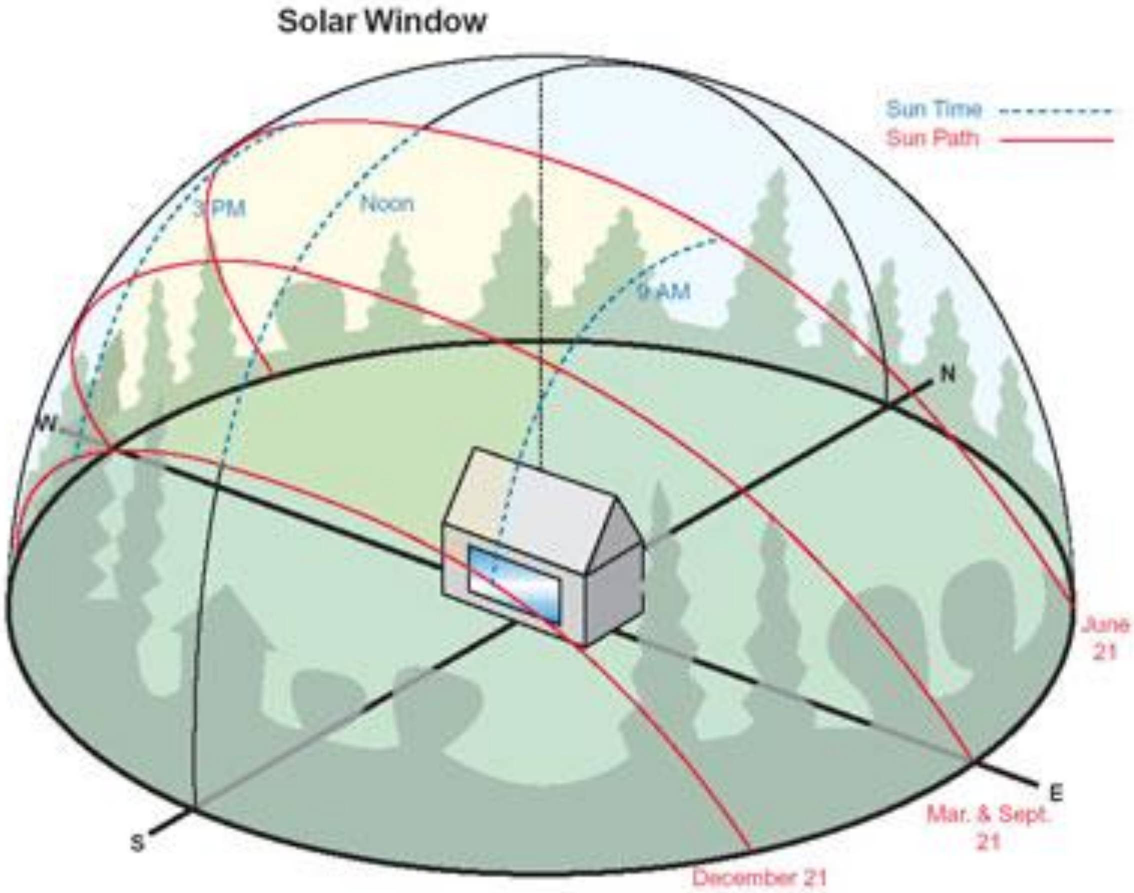
Quiz



NORTH POLE
NORTH AMERICA
New Orleans
ATLANTIC OCEAN
EUROPE
AFRICA
Prime Meridian
SOUTH AMERICA
110°
100°
90°
80°
70°
60°
50°
40°
30°
20°
10°
0°
10°
20°
30°
40°
50°
NORTH POLE
90°
80°
70°
60°
50°
40°
30°
20°
10°
0°
EQUATOR
10°
20°
30°
40°
50°
ASIA
EUROPE
NORTH AMERICA
New Orleans
ATLANTIC OCEAN
AFRICA
SOUTH AMERICA

SECTION III – Solar Position

Predicting the location of the  in the 



A second to think…

What important factors might we have to consider when calculating the location of the sun in the sky?

Quantifying Solar Position

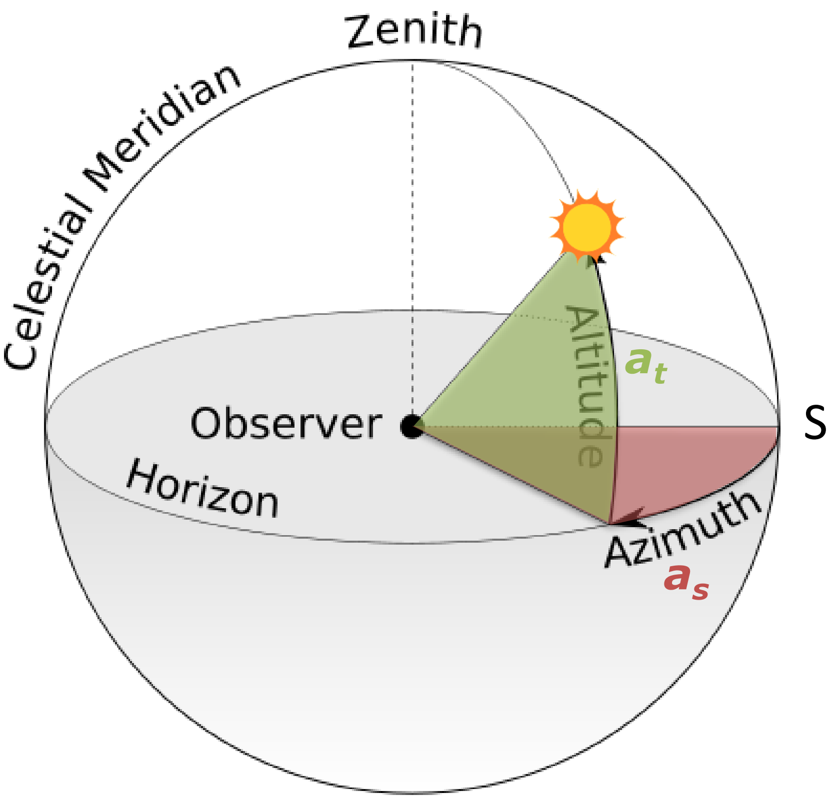
There are two basic quantities that are used for quantifying the location of the sun in the sky. They are:

Solar **Altitude** Angle: a_t

is the vertical *angle* of the sun with respect to the horizon (positive above the horizon)

Solar **Azimuth** Angle: a_s

is the *angle* of the sun – measured in the horizontal plane – relative to south. (west of south is positive (+))



Calculating Solar Position

Before we can calculate a_t and a_s, we need to calculate **Solar Time (*t*)**. Because of many factors, the location of the sun in the sky is not directly related to the time that shows on your watch (which is called **local time**). By using equations that describe the complex geometry between the Earth and the sun, we can calculate the suns location in the sky with a few steps. We need to convert **local time** (time on your watch) to **solar time** (related to the suns position in the sky).

The **first step** is to calculate **Solar Time (*t*)**

This requires ***3 steps*** which are as follows:

1. Calculate **Standard Time** (t_s)
2. Calculate the **Equation of Time** (**ET**)
3. Calculate the **Longitude Correction**

Solar Time – Step 1

The first step in calculating **Solar Time (*t*)** is to calculate **Standard Time (*t_s*)**

What is daylight savings time (DST) is in effect, one hour must be subtracted from the local clock time to arrive at standard time t_s. This is because, in the United States, we change out clocks by one hour in the summer, but this only changes what our watches say. It doesn't move the sun!

Note: In the United States, Daylight Savings Time is in effect beginning the second Sunday in March and ends the first Sunday in November.

If DST is in effect:

$$t_s = t_{local}\textbf{(what your watch reads)} - 1$$

(time is measured on a 24-hour clock)

Solar Time – Step 2

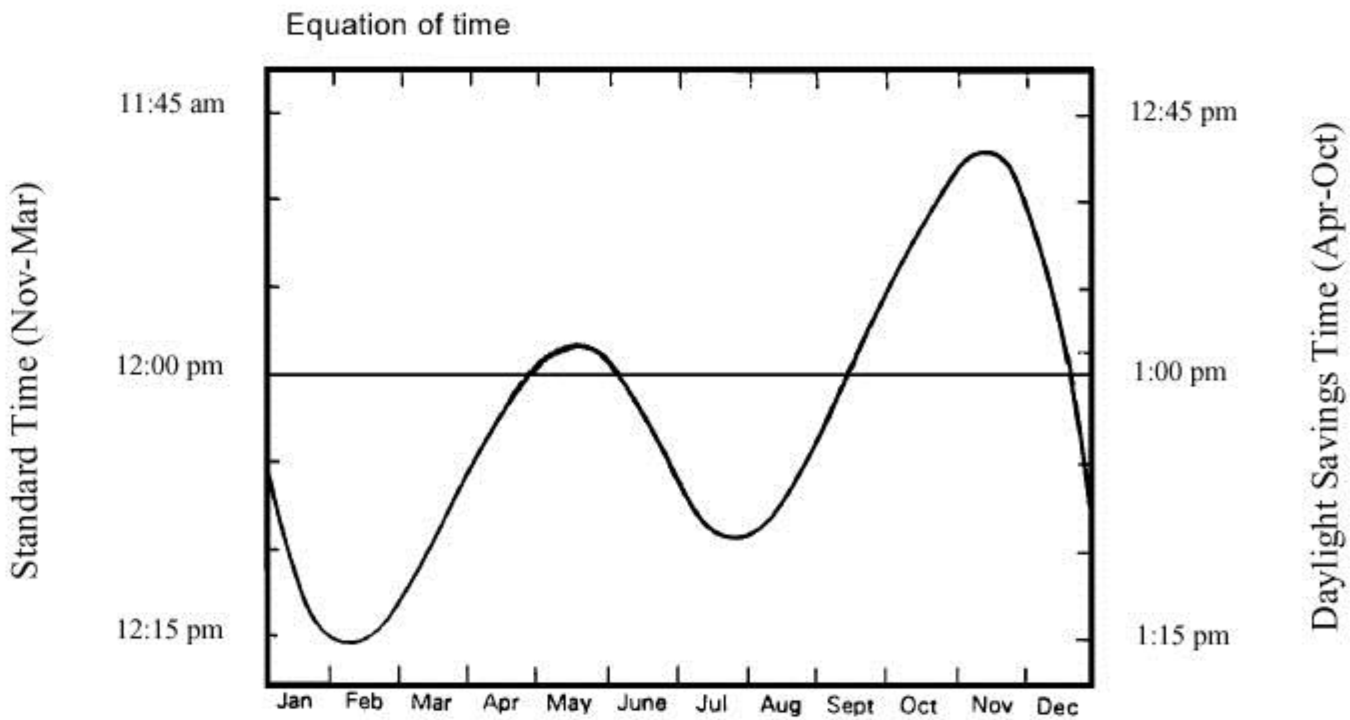
The second step in calculating **Solar Time (*t*)** is to calculate the **Equation of Time (ET)**. The equation of time is used to account for the Earth's elliptical orbit about the sun and the tilt of the Earth's axis relative to its plane of orbit. This equation adjusts the time between -14 minutes and +16 minutes over the year.

$$ET = 0.1644 * Sin\left(\frac{4\pi(J - 81.6)}{365.25}\right) - 0.1273 * Sin\left(\frac{2\pi(J - 2.5)}{365.25}\right)$$

J = Julian day (between 1 and 365)

Solar Time – Step 2 (cont'd)

A graph of the Equation of Time is shown below for reference. More accurate results will come from using the equations directly and not dulling numbers from this graph.



Solar Time – Step 3

The third step in calculating **Solar Time (*t*)** is to calculate the **Longitude Correction**. The longitude correction accounts for your observers longitude relative to a time zones standard meridian (its center longitude). Time zones are nominally 15° wide, therefore solar noon at the east and west boundaries of a time zone occur approximately one-half hour earlier and one-half hour later than at the standard meridian.

$$\boldsymbol{longitude\ correction} = \frac{12 * (SM - L)}{\pi}$$

SM = Standard meridian for the time zone (in radians)
L = Longitude of observer (in radians)

Note that the correction is calculated based on your longitude *relative* to your time zones meridian. When you are located *exactly* on the standard meridian for a time zone (SM = L), then the longitude correction is 0!

Solar Time

Now that you have completed all 3 steps, you can calculate **Solar Time (*t*)**.

$$\boldsymbol{t} = t_s + ET + longitude\ correction$$

$$\boldsymbol{t} = t_s + ET + \frac{12 * (SM - L)}{\pi} \quad \text{(radians)}$$

$$\boldsymbol{t} = t_s + ET + \frac{12 * (SM - L)}{180} \quad \text{(degrees)}$$

t = solar time in decimal hours
t_s = standard time in decimal hours
ET = time from equation of time in decimal hours
SM = standard meridian for the time zone
L = site longitude

Understanding Solar Time

Now that we have the equations for calculating solar time, let's try a bit to understand what it means and how it relates to our watches.

Solar Noon occurs half way through the day, when the sun is the highest in the sky for the day. After solar noon the sun begins to set. This always occurs when the sun falls exactly south in the sky. Put another way, imagine you are standing outside, facing exactly south. The sun will be the highest in the sky when it aligns with your line of sight.

Now, when this happens, what will your watch say? Well, if you are not experiencing daylight savings time ($t_s = t_{local}$), (ET = 0), and you are standing on the standard meridian for your time zone (SM = L), then your clock will read 12pm! If one of these conditions is not true, then your clock will not read 12pm when the sun is highest in the sky (solar noon). For example, if you are experiencing daylight savings time, the difference will be at least an hour, because we change our clocks by 1 hour during DST.

Solar Declination

Once we have calculated our Solar Time, we need to calculate the solar declination (the angle between plane of Earth’s equator and the rays of the sun. This value ranges from the +23.5° (Summer solstice) to -23.5° (Winter Solstice). The solar declination is equal to 0 during both Equinox’s.

$$\boldsymbol{\delta} = 0.4093 * sin\left(\frac{2\pi(J-81)}{368}\right)$$

$\boldsymbol{\delta}$ = solar declination in radians
J = Julian day

Solar Angles – a_t *and* a_s

We now have all of the pieces we need to calculate the suns position in the sky at any given time! The equations are as follows:

$$a_t = \sin^{-1}\left(\sin(l)\sin(\delta) - \cos(l)\cos(\delta)\cos\left(\frac{\pi t}{12}\right)\right)$$

$$a_s = \tan^{-1}\left(\frac{-\cos(\delta)\sin\left(\frac{\pi t}{12}\right)}{-\left(\cos(l)\sin(\delta) + \sin(l)\cos(\delta)\cos\left(\frac{\pi t}{12}\right)\right)}\right)$$

$\boldsymbol{\delta}$ = solar declination in radians
l = observer latitude in radiance
t = solar time in decimal hours

Key Terms

- **Solar Altitude**
- **Solar Azimuth**
- **Daylight Savings Time**
- **Local Time**
- **Standard Time**
- **Equation of Time**
- **Longitude Correction**
- **Solar Declination**

Key Terms (defined)

Solar Altitude (a_t)	is the *vertical angle* of the sun with respect to the horizon (positive above the horizon)
Solar Azimuth (a_s)	is the *angle* of the sun – measured in the horizontal plane – relative to south. (west of south is positive (+))
Daylight Savings Time (DST)	is the practice of advancing clocks during summer months by one hour so that evening daylight lasts an hour longer. Historically, this was to extend the amount of sunlight in the evening to save energy when incandescent lighting was much more prominent. Typically, regions who practice DST adjust clocks forward one hour close to the start of spring and adjust them backward in the autumn to standard time. People use the terms "spring forward" and "fall back" when referring to this.
Local time	The time on your clocks and watches

Key Terms (defined)

Standard time	A time adjusted to compensate for the 1 hour discrepancy between solar time and local time when daylight savings time is in effect.
Equation of Time	describes the discrepancy between two kinds of solar time. The two times that differ are the apparent solar time, which directly tracks the motion of the sun, and mean solar time, which tracks a theoretical "mean" sun with noon's 24 hours apart. Because noon’s on early are not exactly 24 hours apart, we need a correction.
Longitude correction	accounts for your longitude relative to your time zones standard meridian (its center longitude). Time zones are nominally 15° wide, therefore solar noon at the east and west boundaries of a time zone occur approximately one-half hour earlier and one-half hour later than at the standard meridian
Solar declination	the angle between plane of Earth’s equator and the rays of the sun. This value ranges from the +23.5° (Summer solstice) to -23.5° (Winter Solstice). The solar declination is equal to 0 during both Equinox’s.

Worksheet

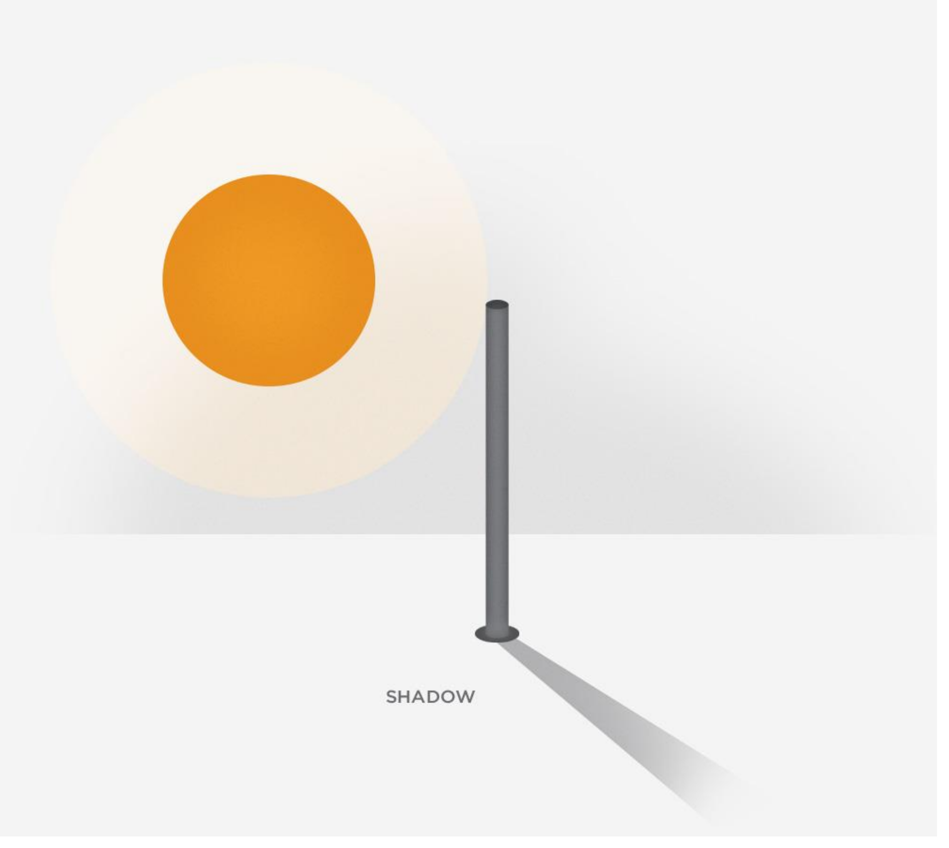
For this worksheet we will calculate the solar altitude and solar azimuth for the sun at a given time.

See worksheet.

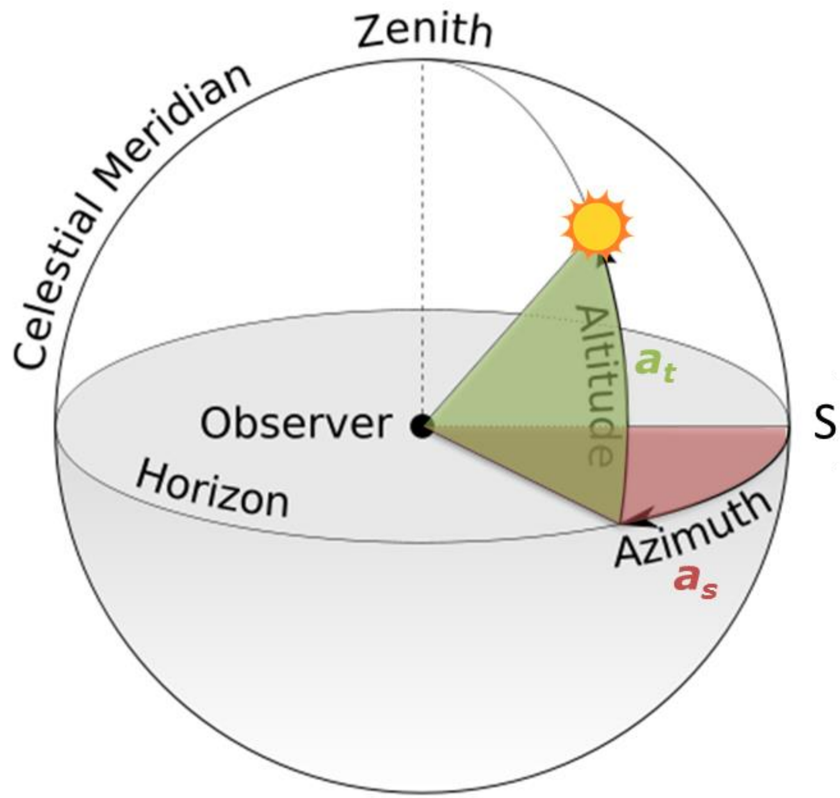
Pick a day that is about a week after this activity is assignment to allow for preparation of the activity.

Activity

For this activity you will use the solar altitude angle and the solar azimuth angle to calculate the shadow of an object outside!



Quiz



Zenith
Celestial Meridian
Altitude
a_t
Observer
S
Horizon
Azimuth
a_s

Stellarium

http://www.stellarium.org/